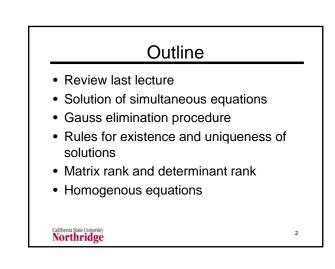
Solution of Simultaneous Linear Equations

Solution of Simultaneous Linear Equations

Larry Caretto Mechanical Engineering 501A Seminar in Engineering Analysis September 11, 2017

California State University Northridge



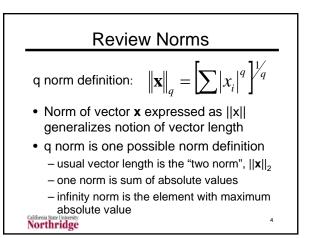
Review Vector Spaces

- Vector spaces are a generalization of the rules for physical vectors
- Have simple rules for elements of vector spaces
- Inner product is a generalization of the dot product
- Norm is a generalization of vector length
- Vector space a unifying concept for many of the topics covered in ME 501AB

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Review Inner Products

- General expression is (x, y)
- For two conventional vectors, [x₁ x₂ x₃ ... x_n] and [y₁ y₂ y₃ ... y_n], the inner product is Σx_iy_i
- For two column vectors, x and y, we can express the inner product as x^Ty
- For two row vectors, x and y, we can express the inner product as xy^T
- We can also define inner products as integrals of two functions Northridge

Review Linear (In)dependence
 A set of vectors *linearly dependent* if the following equation holds, where at least one of the α_i is not equal to zero.
 α₁**x**₍₁₎ + α₂**x**₍₂₎ + … + α_k**x**_(k) = ∑_{i=1}^k α_i**x**_(i) = **0** A *linearly independent* set of vectors is one that is not linearly dependent.
 Cannot have **x**_(i) = **0** in LI set

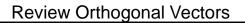
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- An n-dimensional vector space has a set of n linearly independent vectors
- No set of n+1 (or more) linearly independent vectors exist in the space
- Any vector in an n-dimensional space can be represented by a linearly independent combination of n vectors.
- A set of n linearly independent vectors is called a **basis set** and is said to **span the space**

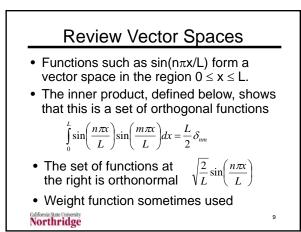
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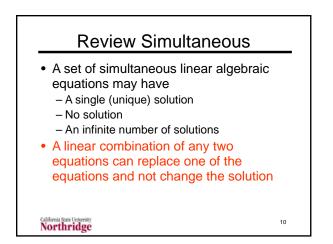
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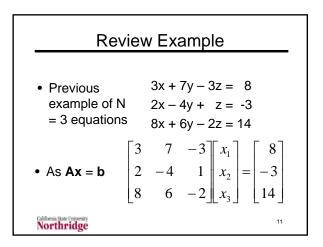


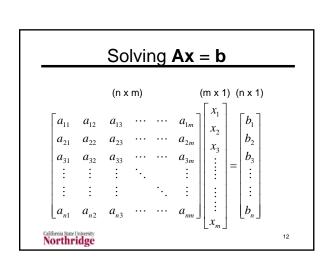
- Two vectors whose inner product equals zero are *orthogonal*.
- A set of n vectors, e₍₁₎, e₍₂₎, ..., e_(n), are mutually orthogonal if (e_(i), e_(i)) = a_iδ_{ij}
- For an **orthonormal** set of vectors, $(\mathbf{e}_{(i)}, \mathbf{e}_{(i)}) = \delta_{ij}$
- The usual unit vectors in mechanics (i, j, and k) are orthonormal

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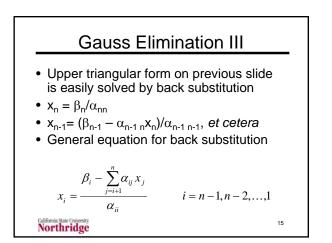


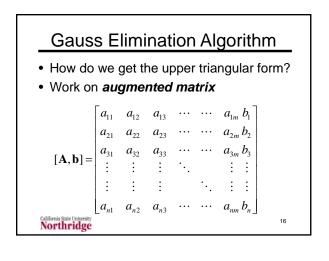


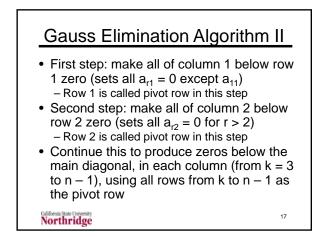


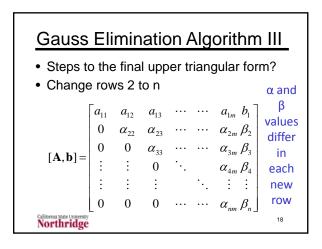
			-	
Gau	uss Elimination			
 Analytical to 	ol for obtaining solutions ol for determining linear or independence			$\begin{bmatrix} \alpha_{11} \\ 0 \end{bmatrix}$
equations (c	s to manipulate the or data) to make them eas nout changing the results	ier		
	Illy create zeros in lower le quations (or data)	eft		: 0 0
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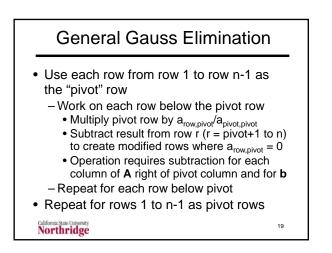
_	ι	Jpp	er 7	[ria	ngula	ar Fo	orm	
•	 Convert original set of equations to 							
$\left[\alpha_{11}\right]$	$\alpha_{\scriptscriptstyle 12}$	$\alpha_{\scriptscriptstyle 13}$			$\alpha_{_{1n-1}}$	α_{1n}	$\begin{bmatrix} x_1 \end{bmatrix}$	$\left[\begin{array}{c} \beta_1 \end{array} \right]$
0	$lpha_{\scriptscriptstyle 22}$	$\alpha_{\scriptscriptstyle 23}$			α_{2n-1}	α_{2n}	<i>x</i> ₂	β_2
0	0	$\alpha_{_{33}}$			$\alpha_{_{3n-1}}$	α_{3n}	x_3	β_3
1	÷	0	·.		÷	:	:	= :
	÷	÷	·.	·.	÷	:	:	
0	0	0		0	α_{n-1n-1}	α_{n-1n}	x_{n-1}	$ \beta_{n-1} $
0	0	0			0	α_{nn}	x_n	β_n
Calif NC	omia State Unit Orthrid	ge						14

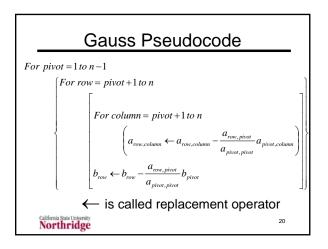


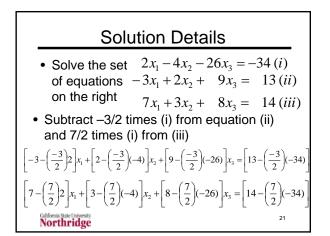


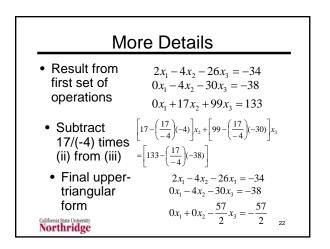


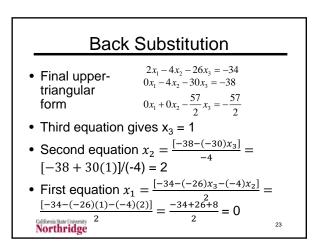


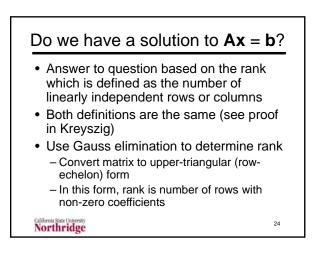


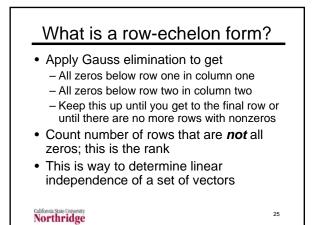


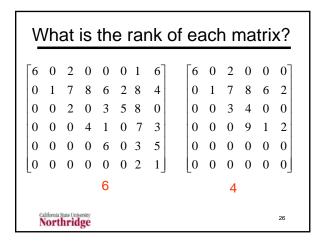


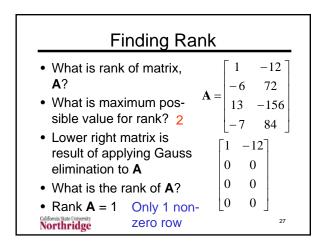












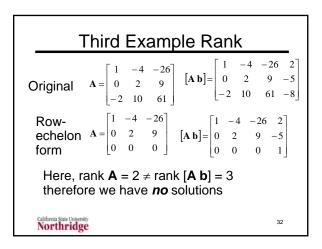
Solutions to Ax = b	
 For a system of n unknowns If rank A = rank [A b] = n there is a unique solution If rank A = rank [A b] < n there are an infinite number of solutions 	
 If rank A ≠ rank [A b] there are no solutions Memorize this! 	28

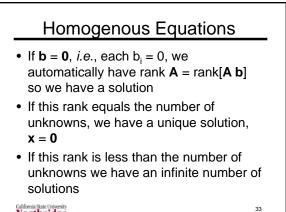
Three	e Examples	
$x_1 - 4x_2 - 26x_3 = 2$	$x_1 - 4x_2 - 26x_3 = 2$	$x_1 = 0$
$2x_2 + 9x_3 = -5$	$2x_2 + 9x_3 = -5$	$x_2 = -7$
$7x_1 + 3x_2 + 8x_3 = -13$	$50.5x_3 = 50.5$	$x_3 = 1$
$x_1 - 4x_2 - 26x_3 = 2$	$x_1 - 4x_2 - 26x_3 = 2$	$x_1 = 12 - 8\alpha$
$2x_2 + 9x_3 = -5$	$2x_2 + 9x_3 = -5$	$x_2 = -2.5 - 4.5\alpha$
$-2x_1 + 10x_2 + 61x_3 = -9$	0 = 0	$x_3 = \alpha$
$x_1 - 4x_2 - 26x_3 = 2$	$x_1 - 4x_2 - 26x_3 = 2$	No
$2x_2 + 9x_3 = -5$	$2x_2 + 9x_3 = -5$	solution
$-2x_1 + 10x_2 + 61x_3 = -8$	0 = 1	00101011
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	$\begin{array}{r} x_1 - 4x_2 - 26x_3 = 2\\ 2x_2 + 9x_3 = -5\\ 7x_1 + 3x_2 + 8x_3 = -13\\ \hline x_1 - 4x_2 - 26x_3 = 2\\ 2x_2 + 9x_3 = -5\\ -2x_1 + 10x_2 + 61x_3 = -9\\ \hline x_1 - 4x_2 - 26x_3 = 2\\ 2x_2 + 9x_3 = -5\\ -2x_1 + 10x_2 + 61x_3 = -8\\ \hline \end{array}$	$\begin{array}{c c} 2x_2 + 9x_3 = -5 \\ 2x_2 + 9x_3 = -5 \\ \hline 2x_1 + 3x_2 + 8x_3 = -13 \\ \hline x_1 - 4x_2 - 26x_3 = 2 \\ 2x_2 + 9x_3 = -5 \\ \hline -2x_1 + 10x_2 + 61x_3 = -9 \\ \hline x_1 - 4x_2 - 26x_3 = 2 \\ 2x_2 + 9x_3 = -5 \\ \hline 2x_1 + 10x_2 + 61x_3 = -8 \\ \hline 0 = 1 \\ \hline \end{array}$

	First Evernale Deals		
	First Example Rank		
Original	$\mathbf{A} = \begin{bmatrix} 1 & -4 & -26 \\ 0 & 2 & 9 \\ 7 & 3 & 8 \end{bmatrix} \begin{bmatrix} \mathbf{A} \ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & -4 & -26 & 2 \\ 0 & 2 & 9 & -5 \\ 7 & 3 & 8 & -13 \end{bmatrix}$		
Row- echelon form	$\mathbf{A} = \begin{bmatrix} 1 & -4 & -26 \\ 0 & 2 & 9 \\ 0 & 0 & 50.5 \end{bmatrix} \qquad \begin{bmatrix} \mathbf{A} \ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & -4 & -26 & 2 \\ 0 & 2 & 9 & -5 \\ 0 & 0 & 50.5 & 50.5 \end{bmatrix}$		
Here we see that rank A = rank [A b] = number of unknowns = 3 so we have a unique solution Northridge			

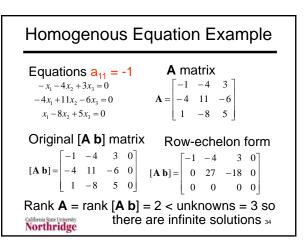
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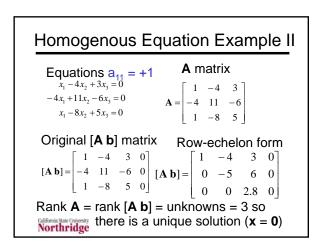
Second Example Rank		
Original $\mathbf{A} = \begin{bmatrix} 1 & -4 & -26 \\ 0 & 2 & 9 \\ -2 & 10 & 61 \end{bmatrix} \begin{bmatrix} \mathbf{A} \ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & -4 & -26 & 2 \\ 0 & 2 & 9 & -5 \\ -2 & 10 & 61 & -9 \end{bmatrix}$		
$\begin{array}{c} \text{Row-} \\ \text{echelon} \\ \text{form} \end{array} \mathbf{A} = \begin{bmatrix} 1 & -4 & -26 \\ 0 & 2 & 9 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \mathbf{A} \ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & -4 & -26 & 2 \\ 0 & 2 & 9 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$		
rank A = rank [A b] = 2 which is less than the number of unknowns (3) so we have an infinite number of solutions Northridge		

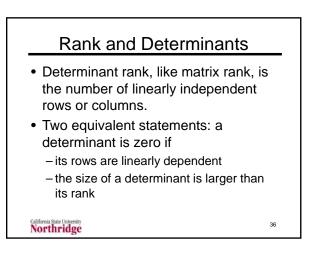


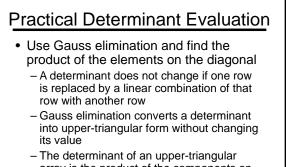












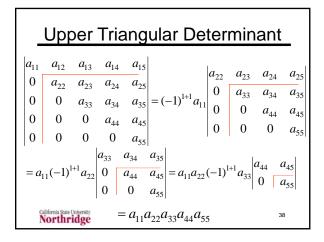
 The determinant of an upper-triangular array is the product of the components on the principal diagonal (example next slide)

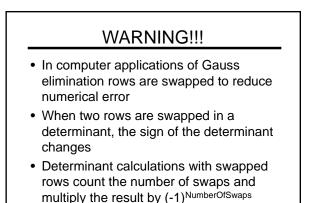
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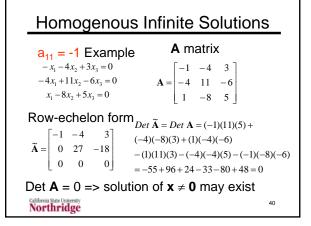
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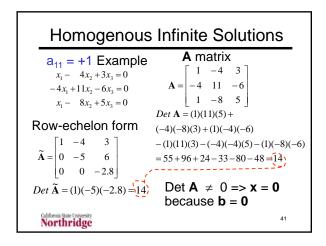
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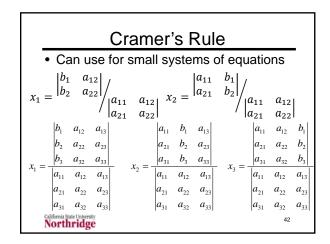
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Rank and Inverses

- Finding A⁻¹ for an n x n matrix requires the solution of Ax = b n times, where b is one column of the unit matrix
- We cannot solve this equation unless rank **A** = n
- An n x n square matrix A will not have an inverse unless its rank equals its size

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- We have previously seen the general result for the elements, b_{ij} , of $\mathbf{B} = \mathbf{A}^{-1}$
- $b_{ij} = C_{ji}/\text{Det}(\mathbf{A})$, where C_{ij} is the cofactor of a_{ij}
- We see that b_{ij} is not defined if Det $\mathbf{A} = 0$
- A⁻¹ does not exist if Det A = 0
- Det **A** = 0 for an n x n determinant shows that Rank **A** < n
- Det A ≠ 0 and rank A = n: two equivalent conditions for A_(n x n) to have an inverse

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